Class XI- MATHEMATICS
Chapter-3 : TRIGONOMETRIC FUNCTIONS
Hand out of Module 1/3

In this module we are going to learn about

- **Angles**
- **Measure of an Angle**
- **Degree measure and Radian measure**
- **Trigonometric Functions**
- **The values of trigonometric functions**

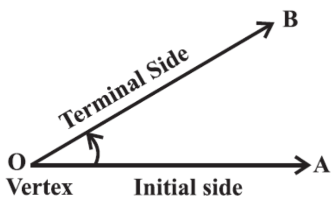
<u>Introduction</u>:

The word ‘trigonometry’ is derived from the Greek words ‘*trigon*’ and ‘*metron*’ and it means ‘measuring the sides of a triangle’. Currently, trigonometry is used in many areas such as in oceanography for calculating the heights of tides in the ocean, in architecture, in medical field, in the science of seismology, designing electric circuits, describing the state of an atom, analysing a musical tone and in many other areas.

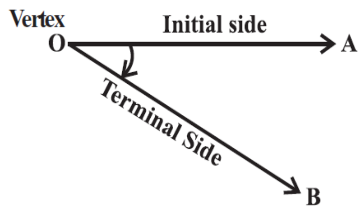
In earlier classes, we have studied the trigonometric ratios of acute angles as the ratio of the sides of a right angled triangle. We have also studied the trigonometric identities and application of trigonometric ratios in solving the problems related to heights and distances. In this Chapter, we will generalise the concept of trigonometric ratios to trigonometric functions and study their properties.

<u>Angles:</u>

Angle is a measure of rotation of a given ray about its initial point. The original ray is called the ***initial side*** and the final position of the ray after rotation is called the ***terminal side*** of the angle. The point of rotation is called the ***vertex*.** If the direction of rotation is anticlockwise, the angle is said to be positive and if the direction of rotation is clockwise, then the angle is ***negative* .**



(i)Positive angle



(ii) Negative angle

<u>Measure of an angle</u>:

The measure of an angle is the amount of rotation performed to get the terminal side from the initial side. There are several units for measuring angles.

The most commonly used units are 1).Degree measure and 2) Radian measure

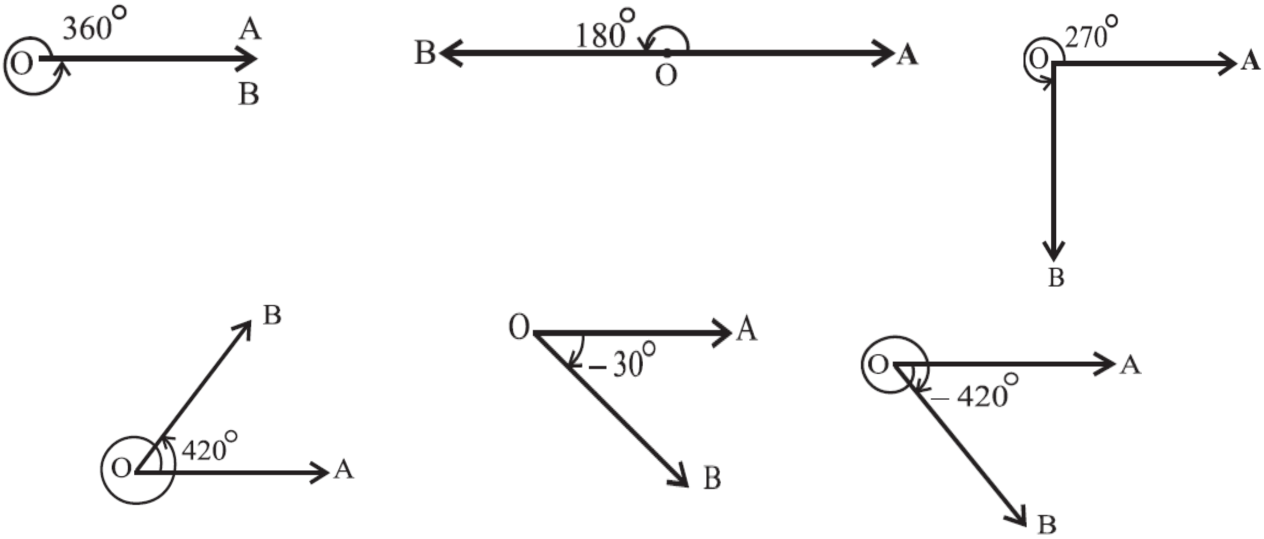
1) **<u>Degree measure</u>** :

If a rotation from the initial side to terminal side is $\left(\frac{1}{360}\right)^{\text{th}}$ of a revolution, the angle is said to have a measure of one degree, written as 1°.

A degree is divided into 60 minutes, and a minute is divided into 60 seconds. One sixtieth of a degree is called a minute, written as $1'$, and one sixtieth of a minute is called a second, written as $1''$.

Thus, $1° = 60'$, $1' = 60''$

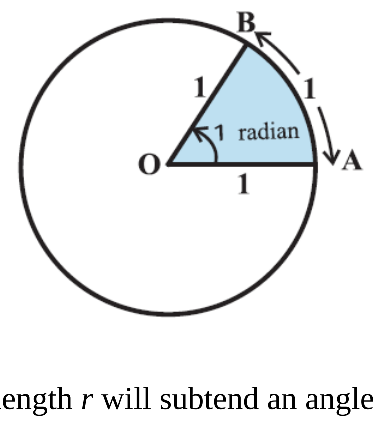
Some of the angles whose measures are 360°, 180°, 270°, 420°, – 30°, – 420° are shown in figure.



2) **<u>Radian measure</u>**:

Angle subtended by an arc of length 1 unit at the centre of a unit circle is said to have a measure of 1 radian. Let OA is the initial side and OB is the terminal side.

The figure shows the angle whose measures is 1 radian.



Hence, in a circle of radius *r*, an arc of length *r* will subtend an angle of 1 radian.

We know that equal arcs of a circle subtend equal angle at the centre.

Thus, in a circle of radius *r*, if an arc of length *l* subtends an angle θ radian at the centre,

Then we have, $\theta = \frac{l}{r}$ or $l = r\,\theta$.

That is length of arc = radius ×central angle (in radian).

Relation between radian and real numbers

Consider the unit circle with centre O. Let A be any point on the circle. Consider OA as initial side of an angle.Then the length of an arc of the circle will give the radian measure of the angle which the arc will subtend at the centre of the circle.

Consider the line PAQ which is tangent to the circle at A. Let the point A represent the real number zero, AP represents positive real number and AQ represents negative real numbers. If we rope the line AP in the anticlockwise direction along the circle, and AQ in the clockwise direction, then every real number will correspond to a radian measure and conversely. Thus, radian measures and real numbers can be considered as one and the same.

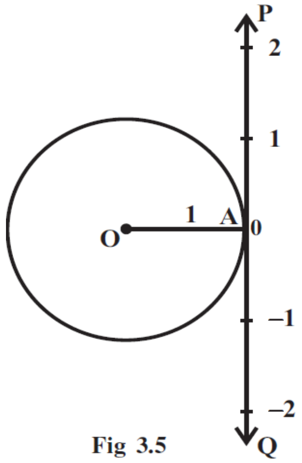


Fig 3.5

Relation between degree measure and radian measure

A circle subtends at the centre an angle whose radian measure is 2π and its degree measure is 360°.

Therefore, 2π radian = 360°

OR

1 radian = $\frac{180°}{\pi}$ = 57° 16′ approximately

1° = $\frac{\pi}{180}$ radian = 0.01746 radian approximately.

Note 1:

The relation between degree measures and radian measure of some common angles are given below:

Degree	30°	45°	60°	90°	180°	270°	360°
Radian	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	π	$\frac{3\pi}{2}$	2π

Note 2: When an angle is expressed in radians, the word 'radian' is frequently omitted.

Thus, π = 180°, $\frac{\pi}{4}$ = 45°

Note 3: Radian measure = $\frac{\pi}{180}$ × Degree measure

Degree measure = $\frac{180°}{\pi}$ × Radian measure

Example 1:

Convert 70° 20′ into radian measure.

$70° \, 20' = 70\frac{1}{3}$ degree $= \frac{211}{3}$ degree $= \frac{211}{3} \times \frac{\pi}{180} = \frac{211\pi}{540}$ radian.

Example 2:

Convert $\frac{5\pi}{3}$ radian into degree measure.

Solution: We know that π radian = 180°.

Therefore, $\frac{5\pi}{3}$ radian = $\frac{180°}{\pi} \times \frac{5\pi}{3}$ = 300°.

Example 3:

The minute hand of a watch is 1.5 cm long. How far does its tip move in 40 minutes? (Use π = 3.14).

Solution: In 60 minutes, the minute hand of a watch completes one revolution.

Therefore in 40 minutes, the minute hand turns through $\frac{2}{3}$ of a revolution.

Therefore, $\theta = \frac{2}{3} \times 2\pi = \frac{4\pi}{3}$ radian.

Hence, the required distance travelled by the minute hand is given by

$l = r\,\theta = 1.5 \times \frac{4\pi}{3}$ cm = 2π cm = 2×3.14 cm = 6.28 cm.

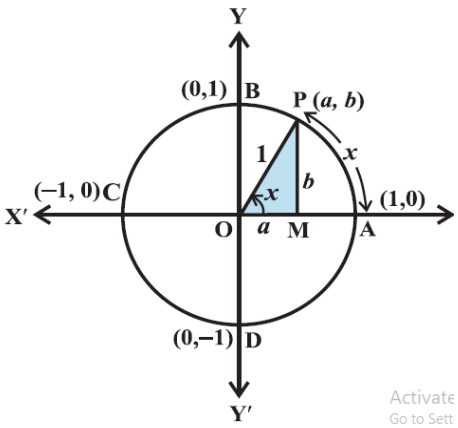
<u>Trigonometric Functions</u>:

Consider a unit circle with centre at origin. Let P (a, b) be any point on the circle with $\angle$ AOP = x radian, i.e., length of arc AP = x. We define cos x = a and sin x = b.

Since $\triangle$OMP is a right triangle, we have, $OM^2 + MP^2 = OP^2$ or $a^2 + b^2 = 1$.

Thus, for every point on the unit circle,

we have $a^2 + b^2 = 1$. That is $\mathbf{\cos^2 x + \sin^2 x = 1}$



Since one complete revolution subtends an angle of 2 π radian at the centre of the circle, $\angle AOB = \frac{\pi}{2}$, $\angle AOC = \pi$ and $\angle AOD = \frac{3\pi}{2}$

All angles which are integral multiples of $\frac{\pi}{2}$ are called quadrantal angles. The coordinates of the points A, B, C and D are respectively (1, 0), (0, 1), (–1, 0) and (0, –1).

Values of sine and cosine functions at quadrantal angles					
Radian (θ)	0	$\frac{\pi}{2}$	π	$\frac{3\pi}{2}$	2π
sin θ	0	1	0	−1	0
cos θ	1	0	-1	0	1

Now, if we take one complete revolution from the point P, we again come back to same point P. Thus, we also observe that if *x* increases (or decreases) by any integral multiple of *2*π, the values of sine and cosine functions do not change. Thus,

sin (2*n*π + *x*) = sin *x*, *n* ∈Z ,

cos (2*n*π + *x*) = cos *x* , *n* ∈Z

Thus

sin *x* = 0 implies *x* = *n*π, and

cos *x* = 0 implies *x* = $(2n+1)\frac{\pi}{2}$**, , where** *n* **∈Z.**

<u>Other trigonometric functions in terms of sine and cosine functions:</u>

tan x = $\frac{\sin x}{\cos x}$, x $\neq (2n+1)\frac{\pi}{2}$, *n* ∈Z.

cot x = $\frac{\cos x}{\sin x}$, x $\neq n\pi$, *n* ∈Z.

cosec x = $\frac{1}{\sin x}$, x $\neq n\pi, n$ ∈ Z.

sec x = $\frac{1}{\cos x}$, x $\neq (2n+1)\frac{\pi}{2}$, *n* ∈Z.

<u>Identities</u>

1) $\cos^2 x + \sin^2 x = 1$

2) $1 + \tan^2 x = \sec^2 x$

3) $1 + \cot^2 x = \mathrm{cosec}^2 x$

<u>The values of trigonometric functions</u>

	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	π	$\frac{3\pi}{2}$	2π
sin	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	1	0	-1	0
cos	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0	-1	0	1
tan	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	not defined	0	not defined	0
cosec	not defined	2	$\sqrt{2}$	$\frac{2}{\sqrt{3}}$	1	not defined	-1	Not defined
sec	1	$\frac{2}{\sqrt{3}}$	$\sqrt{2}$	2	not defined	-1	not defined	1
cot	not defined	$\sqrt{3}$	1	$\frac{1}{\sqrt{3}}$	0	not defined	0	not defined

Example 1:

Find the value of sin $\frac{31\pi}{3}$

Solution: We know that values of sin x repeat after an interval of 2π.

Therefore $\sin \frac{31\pi}{3} = \sin (10\pi + \frac{\pi}{3}) = \sin \frac{\pi}{3} = \frac{\sqrt{3}}{2}$.

Example 2

Prove that $\sin^2 \frac{\pi}{6} + \cos^2 \frac{\pi}{3} - \tan^2 \frac{\pi}{4} = \frac{-1}{2}$

Proof : LHS = $\sin^2 \frac{\pi}{6} + \cos^2 \frac{\pi}{3} - \tan^2 \frac{\pi}{4}$

$$= \left(\frac{1}{2}\right)^2 + \left(\frac{1}{2}\right)^2 - (1)^2$$

$$= \frac{1}{4} + \frac{1}{4} - 1 = \frac{-1}{2} = \text{RHS}$$

Hence proved

Example 3

Evaluate : $\sin^2 \frac{\pi}{4} - \cos \frac{\pi}{6} + 2 \tan^2 \frac{\pi}{3}$

Solution : $\sin^2 \frac{\pi}{4} - \cos \frac{\pi}{6} + 2 \tan^2 \frac{\pi}{3} = \left(\frac{1}{\sqrt{2}}\right)^2 - \frac{\sqrt{3}}{2} + 2\left(\sqrt{3}\right)^2$

$$= \frac{1}{2} - \frac{\sqrt{3}}{2} + 6 = \frac{13 - \sqrt{3}}{2}$$

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